

MONTESSORI CURRICULUM TO STANDARDS ALIGNMENT

PRIMARY • PK3–K

MATHEMATICS

Montessori Curriculum to Standards Alignment
Primary • PK3–K
Mathematics

National Center for Montessori in the Public Sector

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Assessment vocabulary drawn from Marzano Resources free online resource, Basic Vocabulary Terms (marzanoresources.com/media/documents/reproducibles/vocab-common-core/basic-terms-and-phrases.pdf)

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CHAPTER 1

MATHEMATICAL PRACTICE

HUMAN TENDENCIES AND THE MATHEMATICAL MIND

MATHEMATICAL PRACTICE AND MONTESSORI

CCSS Standards for Mathematical Practice

The Common Core State Standards for Mathematics begin with eight Standards for Mathematical Practice (CCSS.MATH.PRACTICE.MP1-8): “processes and proficiencies,” rather than specific grade-level skills and concepts, which mathematics educators at all levels should seek to develop in their students.

These standards are typically grouped into four interrelated clusters:

- Problem Solving
 - MP1: Make sense of problems and persevere in solving them.
 - MP6: Attend to precision.
- Communication: Reasoning and Explaining
 - MP2: Reason abstractly and quantitatively.
 - MP3: Construct viable arguments and critique the reasoning of others.
- Modeling and Using Tools
 - MP4: Model with mathematics
 - MP5: Use appropriate tools strategically.
- Seeing Structures and Generalizing
 - MP7: Look for and make use of structure.
 - MP8: Look for and express regularity in repeated reasoning.

Montessori Mathematics and Geometry

Montessori lessons which support the development of these practices are found in every section of the mathematics and geometry curriculum. The Montessori approach is grounded in a model of human development which explicitly addresses these standards with two concepts: the Human Tendencies and the Mathematical Mind. The Montessori mathematics manipulatives (the “Montessori Materials”) also support the development of these processes and proficiencies through didactic presentation by adults and independent work by children.

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MATHEMATICAL PRACTICE AND MONTESSORI

Human Tendencies

Montessori recognizes basic human behaviors and inclinations and behaviors termed “Human Tendencies”, elaborated in *The Human Tendencies and Montessori Education* by (Montessori, Mario, Association Montessori Internationale, 1956). They are generally summarized as:

- Orientation: Observing one’s environment and building an ordered mental model of the relevant elements.
- Order: Closely related to orientation, the tendency to build an ordered mental model and place material objects, entities, and concepts within it.
- Movement: Physical movement of the body or parts of the body to interact with the environment.
- Exploration: Physical and mental interaction with the environment and with mental models to better understand and develop their qualities.
- Abstraction/Imagination/Reason: Thinking about things not physically present, categorizing, reasoning, and creating new ideas.
- Manipulation: Using the hands or the body to reorder or change elements of the environment.
- Work: Persistent activity to achieve a desired end.
- Exactness: Awareness of the possibility for precision, and activity directed at achieving it.
- Repetition: Repetition, possibly with variation, of an activity or sequence, often with the goal of improving effectiveness.
- Self-Perfection: Activity specifically directed at improving and refining practice.
- Communication: Exchanging information with other humans.

Mathematical Mind

In the Montessori framework, the Human Tendencies operate throughout development and across the entire range of human experience. Montessori also recognizes an innate disposition towards mathematics as the “Mathematical Mind”, a disposition which is present in all human beings but which must be supported and cultivated. The characteristics of the Mathematical Mind (which overlap with the Human Tendencies) are:

- A propensity to learn things which enhance the ability to be exact and orderly, and to observe, compare, and classify.
- Tendencies to calculate, measure, reason, abstract, imagine, and create.

Montessori Materials

The Montessori approach incorporates manipulative materials for mathematics (and other subjects) designed to introduce and reinforce mathematical and geometric concepts through direct instruction and independent exploration. The materials share some common characteristics:

- Attractive and engaging appearance that invites independent exploration.
- Mathematically precise construction.
- Appealing to multiple senses (e.g., touch, color, size, shape, etc.)
- Isolation of a single concept.
- Material representation of abstract concepts.

Montessori PK3-K (Primary) Curriculum and Materials

The Montessori PK3-K (Primary) curriculum and materials are particularly well-designed to support and develop mathematical thinking in young children. Most materials and lessons support order and sequence in their use. Many Sensorial materials are organized in sets of ten to prepare the child for learning the decimal system. Materials, and in particular Sensorial materials, are mathematically precise in their construction, so the gradations between each item are consistent and reinforce numeracy and the decimal system.

MONTESSORI LESSONS

Mathematics

The child explores shape, size, and numeration through the materials.

Practical Life

The order and sequence in the Practical Life exercises prepares the child to reason because they have developed an understanding that steps need to be taken in a particular order.

Sensorial

The Sensorial materials provide children with abstractions. The purpose of the materials is to provide children with the abstraction of color, shape, and size. These materials are referred to as “materialized abstractions.”

Language

The names that are attached to the Sensorial materials which allow the mind to hold the abstraction are given through language. Additionally, spoken language activities support mathematical thinking.

NOTES

References and Resources:

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- O’Connell, S., & SanGiovanni, J. (2013). *Putting the Practices Into Action: Implementing the Common Core Standards for Mathematical Practice, K-8* (Illustrated ed.). Heinemann.
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PROBLEM SOLVING

SKILLS INVENTORY

Lower and Upper Elementary

Solve problems in all areas of the Montessori math curriculum, attending to precision in use of the materials, communication, and calculations, and persevering by trying different approaches and seeking support.

MONTESSORI PRINCIPLES AND SPECIFIC SKILLS

Human Tendencies

Human Tendencies relevant to problem solving include:

- Order
- Exactness
- Abstraction/Imagination/Reason
- Work

Mathematical Mind

Characteristics of the Mathematical Mind to be supported in this area include:

- Order
- Exactness
- Calculation
- Measurement

Specific skills to develop for problem solving include:

Perseverance

- Try multiple strategies if one does not work.
- Seek support from peers or an adult if a problem is unfamiliar or challenging.
- Reflect on similar math experiences when faced with something new.
- Try to visualize the problem and the solution.

Precision

- Be precise:
 - In use of the materials.
 - In communication so others understand what they mean.
 - In calculations.
 - In explanations of what the work or data they are presenting means.
- Use the precise language of mathematics:
 - Use terms accurately.
 - Label work with care.

ASSESSMENT CONSIDERATIONS

Students will be asked to:

- Understand mathematics involves solving problems.
- Try multiple strategies when problem solving.
- Discuss how they solved problems.
- Explain the meaning of a problem.
- Explore ways to solve problems.
- Use concrete objects or pictures to help them conceptualize and solve problems.
- Develop mathematical communication skills.
- Use clear and precise language in discussions with others and in their own reasoning.

Examples:

- Solving addition word problems.
- Solving subtraction word problems.

COMMON CORE STATE STANDARDS (CCSS.MATH.PRACTICE)

MP1	Make sense of problems and persevere in solving them.
MP6	Attend to precision.

COMMUNICATION: REASONING AND EXPLAINING

SKILLS INVENTORY

Demonstrates understanding of the Montessori math curriculum by explaining their own reasoning and responding to the reasoning of others.

MONTESSORI PRINCIPLES AND SPECIFIC SKILLS

Human Tendencies

Human Tendencies relevant to Communication: Reasoning and Explaining include:

- Abstraction/Imagination/Reason
- Communication

Mathematical Mind

Characteristics of the Mathematical Mind to be supported in this area include:

- Reasoning
- Abstraction

Specific skills to develop for Communication: Reasoning and Explaining include:

Reasoning

- Use the Montessori materials or models to explain thinking.
- Explain the numbers and symbols that are being used.
- Relate thinking to a real-world application.

Vocabulary

- Know the correct Montessori, academic, and domain-specific vocabulary.

Collaboration

- Work with classmates on math.

Communication

- Make and defend a mathematical point.
- Justify conclusions by communicating what they think and why they think it.
- Listen and follow other students' reasoning and say if the reasoning makes sense or leaves them with questions.

ASSESSMENT CONSIDERATIONS

Students will be asked to:

- Explain thinking to others.
- Respond to others’ thinking.
- Participate in mathematical discussions involving questions like “How did you get that?” and “Why is that true?”.

Examples:

- Describe objects in the environment using names of shapes.
- Describe the relative positions of objects (above, below, beside, in front of, behind, next to).
- Use informal language to describe similarities, differences, and attributes of two- and three-dimensional shapes.
- Describe measurable attributes of objects.

COMMON CORE STATE STANDARDS (CCSS.MATH.PRACTICE)

MP2	Reason abstractly and quantitatively.
MP3	Construct viable arguments and critique the reasoning of others.

MODELING AND USING TOOLS

SKILLS INVENTORY

Represents math problems in multiple ways such as using Montessori materials, numbers, words, drawing pictures, acting out, making a chart or list, or creating equations.

MONTESSORI PRINCIPLES AND SPECIFIC SKILLS

Human Tendencies

Aspects of the Human Tendencies relevant to Modeling and Using Tools include the tendencies for

- Abstraction/Imagination/Reason
- Manipulation

Mathematical Mind

Characteristics of the Mathematical Mind to be supported In this area include tendencies to:

- Measure
- Reason
- Abstract
- Imagine

Specific activities to support Modeling and Using Tools include:

Montessori Materials

- Use Montessori materials to find solutions.
- As students move towards abstraction, use drawings, tables, charts, graphs, and/or words to find solutions.
- Consider the most efficient tool for the task.

Real-Life Situations

- Apply math to real-life situations.
- Set up a math problem or equation with Montessori materials or on paper.
- Return to the question and explain their answer in relationship to the situation.

ASSESSMENT CONSIDERATIONS

Students will be asked to:

- Experiment with representing problems in multiple ways by using Montessori materials, numbers, words, drawing pictures, acting out, making a chart or list, creating equations, etc.
- Connect different materials and/or representations and explain the connections.
- Consider all available materials and representations when solving a mathematical problem.
- Decide when certain materials or representations might be helpful.
- Compare two materials and/or representations side-by-side.
- Construct arguments using objects, pictures, drawings, and actions.

Examples:

- Montessori math materials.
- Represent a problem by using:
 - objects
 - fingers
 - mental images
 - drawings
 - sounds (claps)
 - acting out situations
 - expressions
 - equations
- Model shapes in geometry with:
 - sticks
 - clay
 - drawings
 - other materials

COMMON CORE STATE STANDARDS (CCSS.MATH.PRACTICE)

MP4	Model with mathematics.
MP5	Use appropriate tools strategically.

SEEING STRUCTURES AND GENERALIZING

SKILLS INVENTORY

Identifies patterns that exist in mathematics and/or with the Montessori materials.

MONTESSORI PRINCIPLES AND SPECIFIC SKILLS

Human Tendencies

Aspects of the Human Tendencies relevant to Seeing Structures and Generalizing include the tendencies for:

- Order
- Abstraction/Imagination/Reason

Mathematical Mind

Characteristics of the Mathematical Mind to be supported in this area include tendencies for:

- Reason
- Abstraction

Specific skills to develop for Seeing Structures and Generalizing include:

Mathematical Structures

- Mathematical structures, including:
 - The number sequence.
 - The place value system.
 - Properties of operations.
 - Formulas for finding area.
 - Inverse relationship of addition and subtraction.
 - Inverse relationship of multiplication and division.
- Recognize similarities and patterns that emerge through repetition.
- Look closely for a pattern or structure.

ASSESSMENT CONSIDERATIONS

Students will be asked to:

- Notice a pattern or structure.
- Notice repetitive actions in counting and computation.
- Check their work by asking themselves, “Does this make sense?”

Examples:

- Montessori material color patterns.
- Recognize the pattern that exists in the teen numbers: the number starts with one ten and ends with the digit that is first stated.
- Recognize $3 + 2$ and $2 + 3$ equal 5.
- Recognize that the next number in a counting sequence is “one more.”

COMMON CORE STATE STANDARDS (CCSS.MATH.PRACTICE)

MP7	Look for and make use of structure.
MP8	Look for and express regularity in repeated reasoning.

CHAPTER 2

THE DECIMAL SYSTEM

NUMBER AND QUANTITY ONE TO TEN

SKILLS INVENTORY

Demonstrates understanding of counting, one-to-one correspondence, and number symbols for 1 to 10.

MONTESSORI LESSONS PURPOSES

NUMBER NAMES AND COUNTING SEQUENCE

Number Rods

- Introduction to Quantity
- Games with Quantities

- To experience the quantities of each number and to learn their respective names.
- To show each number through a single object separate from the others: quantity as a single entity.
- To count objects saying the number names in the standard order, pairing each object with one name, and each number name with one object.
- To memorize the sequence of the numbers 1-10.
- To be sure the child has all the ideas of quantity before introducing symbols.
- To count to answer “how many?” questions.
- To count forward beginning from a given number within the known sequence.
- To connect counting to cardinality.
- To pair each object with one number name and each number name with one object
- To demonstrate understanding that the last number name said tells the number of objects counted.
- To demonstrate understanding that the number of objects is the same regardless of their arrangement or the order in which they were counted.

Sandpaper Numerals

- To introduce the child to the symbols for the numbers s/he already knows.
- To provide a key to the world of written numbers.
- To prepare for writing numerals.
- To say the number names in the standard order.

continues

MONTESSORI LESSONS	PURPOSES
<p>Number Rods and Cards</p> <ul style="list-style-type: none"> • Association of Quantity and Symbol • Distance Game • Arranging in Sequence • Impression of Addition • Impression of Subtraction 	<ul style="list-style-type: none"> • To connect the symbols and the concrete representation of quantity and show the child they are the same. • To compare two numbers between 1 and 10 presented as written numerals. • To be able to demonstrate that successive number names refer to a quantity that is one larger. • To be able to understand that the last number name said tells the number of objects counted. • To see the hierarchical order 1-10 exactly laid out. • To count forward beginning from a given number within the known sequence. • To represent a number of objects with a written numeral 0-10. • To demonstrate understanding of the relationship between numbers and quantities. • To show concrete examples of ways to add quantities. • To connect counting to cardinality. • To say the number names in the standard order. • To pair each object with one number name and each number name with one object. • To demonstrate understanding that the last number name said tells the number of objects counted. • To demonstrate understanding that the number of objects is the same regardless of their arrangement or the order in which they were counted. • To demonstrate understanding that each successive number name refers to a quantity that is one larger. • Precursor to addition and subtraction.
<p>Spindle Boxes</p>	<ul style="list-style-type: none"> • To clarify quantity as a set of separate objects. • To clarify that symbols represent a certain quantity of separate objects. • To understand that each successive number name refers to a quantity that is one larger. • To introduce zero as the empty set and its symbol. • To reinforce the sequence of the numerals 0-9. • To reinforce the idea that 0-9 are the only numerals in our system of numeration. • To represent a number of objects with a written numeral 0-9. • To use 0 to represent a count of no objects.

continues

MONTESSORI LESSONS	PURPOSES
<p>Spindle Boxes</p>	<ul style="list-style-type: none"> • To demonstrate understanding of the relationship between numbers and quantities. • To connect counting to cardinality. • To say the number names in the standard order. • To demonstrate understanding that the last number name said tells the number of objects counted. • To demonstrate understanding that the number of objects is the same regardless of their arrangement or the order in which they were counted. • To demonstrate understanding that each successive number name refers to a quantity that is one larger.
<p>Zero Game</p>	<ul style="list-style-type: none"> • To provide a concrete experience that zero is a placeholder for nothing. • To write numbers from 0 to 20. • To represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects). • To use 0 to represent a count of no objects.
<p>Cards and Counters</p> <ul style="list-style-type: none"> • Introduction • Odds and Evens 	<ul style="list-style-type: none"> • To reinforce the knowledge that each number is made up of separate quantities. • To verify the sequence of numbers from 1-10. • To verify the quantities that correspond to each symbol. • To count objects saying the number names in the standard order, pairing each object with one number name and one object. • To indicate the odd and even numbers. • To see the concrete difference between odds and evens. • Preparation for the divisibility of numbers. • To represent a number of objects with a written numeral 0-10. • To demonstrate understanding of the relationship between numbers and quantities. • To connect counting to cardinality. • To say the number names in the standard order. • To demonstrate understanding that the last number name said tells the number of objects counted. • To demonstrate understanding that the number of objects is the same regardless of their arrangement or the order in which they were counted. • To demonstrate understanding that each successive number name refers to a quantity that is one larger.

continues

MONTESSORI LESSONS	PURPOSES
Bead Stair	<ul style="list-style-type: none"> • To associate numerals with bead bars representing quantities 0–10 • To practice linear counting. • To represent a number of objects with a written numeral 0–10. • To demonstrate understanding of the relationship between numbers and quantities. • To connect counting to cardinality. • To say the number names in the standard order. • To pair each object with one number name and each number name with one object. • To demonstrate understanding that the last number name said tells the number of objects counted. • To demonstrate understanding that the number of objects is the same regardless of their arrangement or the order in which they were counted. • To demonstrate understanding that each successive number name refers to a quantity that is one larger.
Memory Game of Numbers	<ul style="list-style-type: none"> • To develop the power of memory to hold a numeric symbol in the mind. • To help the child transfer knowledge of the numbers from 0-10 from the specific materials to the objects of daily life. • To understand that the last number name said tells the number of objects counted. • To represent a number of objects with a written numeral 0-10. • To demonstrate understanding of the relationship between numbers and quantities. • To connect counting to cardinality. • To demonstrate understanding that the last number name said tells the number of objects counted. • To demonstrate understanding that the number of objects is the same regardless of their arrangement or the order in which they were counted.
Number Writing <ul style="list-style-type: none"> • Teacher-Created Lessons 	<ul style="list-style-type: none"> • To write numbers from 0–10.

continues

MONTESSORI LESSONS		PURPOSES
Counting 0–20 Objects • Teacher-Created Lessons	<ul style="list-style-type: none"> • To answer “how many?” questions by counting up to 20 things arranged in a line, a rectangular array, or a circle. • To count out that number of objects when given a number from 1-20. 	
Counting 0–10 Objects • Teacher-Created Lessons	<ul style="list-style-type: none"> • To answer “how many?” questions by counting up to 10 things in a scattered configuration. 	
COMPARING NUMBERS		
Greater Than, Less Than, Equal To • Teacher-Created Lessons	<ul style="list-style-type: none"> • To compare 2 set of objects as greater than, less than, or equal to. • To identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group. 	
Compare Quantity of Written Numerals • Teacher-Created Lessons	<ul style="list-style-type: none"> • To compare two written numerals between 1 and 10 presented as written numerals. 	

ASSESSMENT VOCABULARY		
array	line	Cognitive Verbs
circle	number	
compare	number name	
count	number order	
equal	ones	
greater than (>)	pair	
group/grouping	quantity	
how many	rectangular array	
large/larger	represent	
number name	zero	
less than (<)		

ASSESSMENT CONSIDERATIONS

Students will be asked to:

Number Names and Counting Sequence

- Begin counting forward from any number (within the known sequence). (K.CC.A.2)
- Write numbers from 0 to 10. (K.CC.A.3)
- Represent a number of objects with a written numeral 0-10. (K.CC.A.3)
- Use 0 to represent a count of no objects. (K.CC.A.3)
- Demonstrate understanding of the relationship between numbers and quantities. (K.CC.B.4)
- Connect counting to cardinality. (K.CC.B.4)
- Say the number names in the standard order. (K.CC.B.4.A)
- Pair each object with one number name and each number name with one object. (K.CC.B.4.A)
- Demonstrate understanding that the last number name said tells the number of objects counted. (K.CC.B.4.B)
- Demonstrate understanding that the number of objects is the same regardless of their arrangement or the order in which they were counted. (K.CC.B.4.B)
- Demonstrate understanding that each successive number name refers to a quantity that is one larger. (K.CC.B.4.C)
- Answer “how many?” questions by counting up to 20 things arranged in a line, a rectangular array, or a circle. (K.CC.B.5)
- Answer “how many?” questions by counting up to 10 things in a scattered configuration. (K.CC.B.5)
- Count out that number of objects when given a number from 1-20. (K.CC.B.5)

Comparing Numbers

- Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group (matching and counting strategies). (K.CC.C.6)
- Compare two numbers presented as written numerals (between 1 and 10). (K.CC.C.7)

COMMON CORE STATE STANDARDS (CCSS.MATH.CONTENT)

K.CC.A.2	Count forward beginning from a given number within the known sequence (instead of having to begin at 1).
K.CC.A.3	Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects).
K.CC.B.4	Understand the relationship between numbers and quantities; connect counting to cardinality.
K.CC.B.4.A	When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object.
K.CC.B.4.B	Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.

continues

COMMON CORE STATE STANDARDS (CCSS.MATH.CONTENT)	
K.CC.B.4.C	Understand that each successive number name refers to a quantity that is one larger.
K.CC.B.5	Count to answer “how many?” questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1-20, count out that many objects.
K.CC.C.6	Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies.
K.CC.C.7	Compare two numbers between 1 and 10 presented as written numerals.

HEAD START EARLY LEARNING OUTCOMES FRAMEWORK (HELOF) GOALS		
DEVELOPMENTAL PROGRESSION		INDICATORS
36 TO 48 MONTHS	48 TO 60 MONTHS	BY 60 MONTHS
MATHEMATICS DEVELOPMENT (P-MATH)		
COUNTING AND CARDINALITY		
P-MATH 1. Child knows number names and the count sequence.		
Says or signs some number words in sequence (up to 10), starting with one. Understands that counting words are separate words, such as “one,” “two,” “three” versus “onetwothree”.	Says or signs more number words in sequence.	<ul style="list-style-type: none"> Counts verbally or signs to at least 20 by ones.
P-MATH 2. Child recognizes the number of objects in a small set		
Develops an understanding of what whole numbers mean. Begins to recognize the number of small objects in groups without counting (referred to as “subitizing”).	Quickly recognizes the number of objects in a small set (referred to as “subitizing”).	<ul style="list-style-type: none"> Instantly recognizes, without counting, small quantities of up to 5 objects and says or signs the number.

continues

HEAD START EARLY LEARNING OUTCOMES FRAMEWORK (HELOF) GOALS**P-MATH 3. Child understands the relationship between numbers and quantities.**

Begins to coordinate verbal counting with objects by pointing to or moving objects for small groups of objects laid in a line (referred to as one-to-one correspondence). Begins to understand that the last number represents how many objects are in a group (referred to as “cardinality”).

Understands that number words refer to quantity. May point to or move objects while counting objects to 10 and beyond (one-to-one correspondence). Understands that the last number represents how many objects are in a group (cardinality).

- When counting objects, says or signs the number names in order, pairing one number word that corresponds with one object, up to at least 10.
- Counts and answers “How many?” questions for approximately 10 objects.
- Accurately counts as many as 5 objects in a scattered configuration.
- Understands that each successive number name refers to a quantity that is one larger.
- Understands that the last number said represents the number of objects in a set.

P-MATH 4. Child compares numbers.

Begins to accurately count and compare objects that are about the same size and are in small groups with adult assistance, such as counts a pile of 2 blocks and a pile of 4, and determines whether the piles have the same or different numbers of blocks. Identifies the first and second objects in a sequence.

Counts to determine and compare number amounts even when the larger group’s objects are smaller in size, such as buttons, compared with the smaller group’s objects that are larger in size, such as markers. Uses numbers related to order or position.

- Identifies whether the number of objects in one group is more than, less than, or the same as objects in another group for up to at least five objects.
- Identifies and uses numbers related to order or position from first to tenth.

P-MATH 5. Child associates a quantity with written numerals up to 5 and begins to write numbers.

Begins to understand that a written numeral represents a quantity and may draw objects or use informal symbols to represent numbers.

Understands that written numbers represent quantities of objects, and uses information symbols, such as a tally, to represent numerals. With adult support, writes some numerals up to 10.

- Associates a number of objects with a written numeral 0–5.
- Recognizes and, with support, writes some numerals up to 10.

NUMBER AND QUANTITY GREATER THAN TEN

SKILLS INVENTORY

- Demonstrates understanding of counting, one-to-one correspondence, and number symbols for 1 to 1,000.
- Demonstrates understanding of the concept of units, tens, hundreds, and thousands by recognizing number patterns that recur across hierarchy families (simple, family of thousands, family of millions).
- Demonstrates understanding of the function of zero as a placeholder and 9 as a counting limit.

MONTESSORI LESSONS

PURPOSES

NUMBER NAMES AND COUNTING SEQUENCE

Golden Beads

- Introduction to Quantity
- Games with Quantities

- To introduce the names for quantities in each category (“unit”; “ten”; “hundred”; “thousand”).
- To show the relationship between one category and the next.
- To offer the child the sensorial experience of the relative sizes of the categories (bulk).
- To extend the sensorial experience of the different categories and the difference in bulk, for instance, between 6 units and 6 hundreds.
- To understand that each successive number name refers to a quantity that is one larger.
- To count to answer “how many?”

Number Cards

- Introduction to Symbol
- Games with Symbols

- To show the relationship between one category and the next.
- To offer the child the sensorial experience of the relative sizes of the categories (bulk).
- To introduce the symbols for the quantities in the decimal system.
- To reinforce language of the categories.
- To introduce the symbols for multiple 1000s, 100s, 10s, 1s.

continues

MONTESSORI LESSONS	PURPOSES
<p>Beads and Cards</p> <ul style="list-style-type: none"> • Association of Quantity and Symbol • Formation of Numbers • Formation of Numbers with Zero • Tray of Nine 	<ul style="list-style-type: none"> • To associate the concrete representation and symbolic representation of the categories. • To introduce the names for quantities in each category (“unit”; “ten”; “hundred”; “thousand”). • To show the relationship between one category and the next. • To offer the child the sensorial experience of the relative sizes of the categories (bulk). • To extend the sensorial experience of the different categories and the difference in bulk, for instance, between 6 units and 6 hundreds. • To introduce the symbols for the quantities in the decimal system. • To understand the relationship between numbers and quantities. • To represent a number of objects with a written numeral 0-20. • To ensure the child knows that zero can hold a place for a category. • To represent a number of objects with a written numeral 0-20. • To demonstrate understanding of the relationship between numbers and quantities. • To connect counting to cardinality. • To say the number names in the standard order. • To pair each object with one number name and each number name with one object.
<p>Bead Cabinet</p> <ul style="list-style-type: none"> • 100 Chain • Short Chains • 1000 Chain • Long Chains • Skip Counting • Recording the Chains • Exploring the Chains • Number Pyramid • Exploration with the Geometric Cabinet 	<ul style="list-style-type: none"> • To consolidate linear counting. • To count to 100 by ones and by tens. • To give a sensorial impression of the difference between 102 and 103. • To solidify the idea that 100 is 10 tens, and 10 hundreds is 1000. • To compare the squares and cubes of the numbers 1-10. • To give the child another means of counting in a series. • To count forward beginning from a given number within the known sequence. • To understand that the number of objects is the same regardless of their arrangement or the order in which they were counted. • To practice writing numerals. • To develop the motivation to repeatedly count the long and short chains. • Indirect preparation for multiplication, squaring, cubing, and bases other than base ten. • To demonstrate understanding of the relationship between numbers and quantities. • To say the number names in the standard order. • To pair each object with one number name and each number name with one object.

continues

MONTESSORI LESSONS	PURPOSES
COMPARING NUMBERS	
<p>Teens Boards</p> <ul style="list-style-type: none"> • Introduction to Quantity • Games with Quantities • Introduction to Symbols • Games with Symbols • Association of Quantity and Symbol 	<ul style="list-style-type: none"> • To introduce the quantities 11-19 with their names. • To see the relationship of the numbers 1-9 to the number 10. • To give the symbols for the numbers 11-19. • To help the child become secure with the symbols for the numbers 11-19. • To associate the quantity, name, and symbol for the teens (11-19). • To represent a number of objects with a written numeral 0-20. • To compose and decompose numbers from 11 to 19 into ten ones and some further ones by using objects or drawings and record each composition or decomposition by a drawing or equation. • To represent a number of objects with a written numeral 0-20. • To demonstrate understanding of the relationship between numbers and quantities. • To say the number names in the standard order. • To pair each object with one number name and each number name with one object. • To demonstrate understanding that the number of objects is the same regardless of their arrangement or the order in which they were counted. • To count out that number of objects when given a number from 1-20.
<p>Tens Boards</p> <ul style="list-style-type: none"> • Tens • Naming the Tens • Counting from 10-99 	<ul style="list-style-type: none"> • To introduce the conventional names for the tens quantities: “twenty”, “thirty”, “forty”...“ninety”. • To realize how the numbers progress from one ten to the next. • To connect name, quantity, and symbol for the numbers 11-99. • To represent a number of objects with a written numeral 0-20. • To demonstrate understanding of the relationship between numbers and quantities. • To say the number names in the standard order. • To pair each object with one number name and each number name with one object. • To demonstrate understanding that the number of objects is the same regardless of their arrangement or the order in which they were counted.
<p>Numeral Writing</p> <ul style="list-style-type: none"> • Teacher-Created Lessons 	<ul style="list-style-type: none"> • To write numbers from 0 to 20.

continues

MONTESSORI LESSONS	PURPOSES
Greater Than, Less Than, equal To • Teacher-Created Lessons	• To compare 2 set of objects as greater than, less than, or equal to.
Compare Quantity of Written Numerals • Teacher-Created Lessons	• To compare two written numerals.

ASSESSMENT VOCABULARY		
array	less than (<)	Cognitive Verbs answer arrange compose decompose Identify record refer represent understand
circle	line	
compose	number	
count	number name	
decompose/decomposition	number order	
equal	ones	
equation	pair	
greater than (>)	quantity	
group/grouping	rectangular array	
how many	represent	
large/larger	zero	

ASSESSMENT CONSIDERATIONS**Students will be asked to:****Number Names and Counting Sequence**

- Count to 100 by ones. (K.CC.A.1)
- Count to 100 by tens. (K.CC.A.1)
- Begin counting forward from any number (within the known sequence). (K.CC.A.2)
- Write numbers from 0 to 20. (K.CC.A.3)
- Represent a number of objects with a written numeral 0-20. (K.CC.A.3)
- Use 0 to represent a count of no objects. (K.CC.A.3)
- Demonstrate understanding of the relationship between numbers and quantities. (K.CC.B.4)
- Connect counting to cardinality. (K.CC.B.4)
- Say the number names in the standard order. (K.CC.B.4.A)
- Pair each object with one number name and each number name with one object. (K.CC.B.4.A)
- Demonstrate understanding that the last number name said tells the number of objects counted. (K.CC.B.4.B)
- Demonstrate understanding that the number of objects is the same regardless of their arrangement or the order in which they were counted. (K.CC.B.4.B)
- Demonstrate understanding that each successive number name refers to a quantity that is one larger. (K.CC.B.4.C)
- Answer “how many?” questions by counting up to 20 things arranged in a line, a rectangular array, or a circle. (K.CC.B.5)
- Answer “how many?” questions by counting up to 10 things in a scattered configuration. (K.CC.B.5)
- Count out that number of objects when given a number from 1-20. (K.CC.B.5)

Comparing Numbers

- Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group (matching and counting strategies). (K.CC.C.6)
- Compose and decompose numbers from 11 to 19 into tens and ones (using objects or drawings). (K.NBT.A.1)
- Record composition or decomposition of numbers by a drawing or equation. (K.NBT.A.1)
- Demonstrate understanding that these numbers are composed of tens and ones. (K.NBT.A.1)

COMMON CORE STATE STANDARDS (CCSS.MATH.CONTENT)

K.CC.A.1	Count to 100 by ones and by tens.
K.CC.A.2	Count forward beginning from a given number within the known sequence (instead of having to begin at 1).
K.CC.A.3	Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects).
K.CC.B.4	Understand the relationship between numbers and quantities; connect counting to cardinality.
K.CC.B.4.A	When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object.
K.CC.B.4.B	Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.
K.CC.B.4.C	Understand that each successive number name refers to a quantity that is one larger.
K.CC.B.5	Count to answer “how many?” questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1-20, count out that many objects.
K.CC.C.6	Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies.
K.NBT.A.1	Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (such as $18 = 10 + 8$); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.

HEAD START EARLY LEARNING OUTCOMES FRAMEWORK (HELOF) GOALS

DEVELOPMENTAL PROGRESSION		INDICATORS
36 TO 48 MONTHS	48 TO 60 MONTHS	BY 60 MONTHS
MATHEMATICS DEVELOPMENT (P-MATH)		
COUNTING AND CARDINALITY		
P-MATH 1. Child knows number names and the count sequence.		
Says or signs some number words in sequence (up to 10), starting with one. Understands that counting words are separate words, such as “one,” “two,” “three” versus “onetwothree”.	Says or signs more number words in sequence.	• Counts verbally or signs to at least 20 by ones.

CHAPTER 3

WHOLE NUMBER OPERATIONS

OPERATIONS

SKILLS INVENTORY

Performs all four operations with 4-digit numbers using Montessori materials.

MONTESSORI LESSONS	PURPOSES
GOLDEN BEADS	
Addition <ul style="list-style-type: none">• Static• Change Game• Dynamic	<ul style="list-style-type: none">• To show that 10 of a category equals 1 of the next higher category: the dynamics of the decimal system.• To understand the role of changing from one category to the next.• To give the sensorial impression of addition: putting quantities together to form a larger quantity.• To practice the role of changing from one category to the next.• To represent addition with objects, drawings, verbal explanations, expressions, or equations.
Multiplication <ul style="list-style-type: none">• Static• Dynamic	<ul style="list-style-type: none">• To give the sensorial impression of multiplication: putting together equal quantities to form a larger quantity.• To show that multiplication is the addition of equal quantities.
Subtraction <ul style="list-style-type: none">• Static• Dynamic	<ul style="list-style-type: none">• To give the sensorial impression of subtraction: taking away smaller quantities from a larger quantity.• To further the understanding of the dynamics of the decimal system.• To reinforce the role of changing from one category to the next—from higher to lower category.• To represent subtraction with objects, drawings, verbal explanations, expressions, or equations.

continues

MONTESSORI LESSONS	PURPOSES
<p>Division</p> <ul style="list-style-type: none"> • Short Division • Long Division • 2-Digit Divisor 	<ul style="list-style-type: none"> • To give the sensorial impression of division: sharing a quantity into a certain number of equal parts. • To show how quantities are distributed in long division. • To show that the answer in division is the amount that one of the units in the divisor receives. • To reinforce the dynamics of the decimal system. • To reinforce the role of changing from one category to the next – from higher to lower category.
STAMP GAME	
<p>Addition</p> <ul style="list-style-type: none"> • Introduction • Static • Dynamic 	<ul style="list-style-type: none"> • To reinforce and consolidate, by means of individual work, the understanding of addition acquired through the collective exercises. • To further the sensorial experience of place value. • To begin to write compound numbers (quantities of more than one category). • To apply the symbol (+) for addition to the decimal system.
<p>Multiplication</p> <ul style="list-style-type: none"> • Static • Dynamic 	<ul style="list-style-type: none"> • To reinforce and consolidate, by means of individual work, the understanding of multiplication acquired through the collective exercises. • To further the sensorial experience of place value. • To begin to write compound numbers (quantities of more than one category). • To apply the symbols for multiplication (x) to the decimal system. • To show the child that multiplication is just adding the same number a certain number of times.
<p>Subtraction</p> <ul style="list-style-type: none"> • Static • Dynamic 	<ul style="list-style-type: none"> • To reinforce and consolidate, by means of individual work, the understanding of subtraction acquired through the collective exercises. • To further the sensorial experience of place value. • To begin to write compound numbers (quantities of more than one category). • To apply the symbol (-) for subtraction to the decimal system. • The way the stamps are used shows the children that subtraction is the opposite of addition.

continues

MONTESSORI LESSONS	PURPOSES
<p>Division</p> <ul style="list-style-type: none"> • Short Division • 2-Digit Divisor • 3-4-Digit Divisor • 0 in the Middle of the Divisor • 0 in the Unit of the Divisor 	<ul style="list-style-type: none"> • To reinforce and consolidate, by means of individual work, the understanding of division acquired through the collective exercises. • To further the sensorial experience of place value. • To begin to write compound numbers (quantities of more than one category). • To apply the symbols for division (\div) to the decimal system. • To show the child that division is the opposite of multiplication. • To practice long division and understand what to do when there is a zero in the units place of the divisor.

ASSESSMENT VOCABULARY		
addition equation expression	represent subtraction	<p>Cognitive Verbs</p> act out represent

ASSESSMENT CONSIDERATIONS
<p>Students will be asked to:</p> <ul style="list-style-type: none"> • Represent addition with objects (Montessori materials), fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations. (K.OA.A.1) • Represent subtraction with objects (Montessori materials), fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations. (K.OA.A.1)

COMMON CORE STATE STANDARDS (CCSS.MATH.CONTENT)	
<p>K.OA.A.1</p>	Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (claps), acting out situations, verbal explanations, expressions, or equations.

HEAD START EARLY LEARNING OUTCOMES FRAMEWORK (HELOF) GOALS

DEVELOPMENTAL PROGRESSION		INDICATORS
36 TO 48 MONTHS	48 TO 60 MONTHS	BY 60 MONTHS
MATHEMATICS DEVELOPMENT (P-MATH)		
OPERATIONS AND ALGEBRAIC THINKING		
P-MATH 6. Child understands addition as adding to and understands subtraction as taking away from.		
<p>Begins to add and subtract very small collections of objects with adult support. For example, the teacher says, “You have 3 grapes and get 1 more. How many in all?” Child counts out 3, then counts out 1 more, then counts all 4: “1, 2, 3, 4. I have 4!”</p>	<p>Solves addition problems by joining objects together and subtraction problems by separating, using manipulatives and fingers to represent objects.</p>	<ul style="list-style-type: none"> • Represents addition and subtraction in different ways, such as with fingers, objects, and drawings. • Solves addition and subtraction word problems. Adds and subtracts up to 5 to or from a given number. • With adult assistance, begins to use counting on from the larger number for addition. For example, when adding a group of 3 and a group of 2, counts “One, two, three...” and then counts on “Four, five!” (keeping track with fingers). When counting back for subtraction such as taking away 3 from 5, counts, “Five, four, three...two!” (keeping track with fingers).

PASSAGES TO ABSTRACTION

SKILLS INVENTORY

Demonstrates understanding of one-to-one correspondence, place value, numeration, and operations using Montessori materials and more abstract methods to solve an equation.

MONTESSORI LESSONS	PURPOSES
Dot Game <ul style="list-style-type: none"> • One Number at a Time • Column Addition 	<ul style="list-style-type: none"> • To focus on the mechanism of carrying numbers up to the next category. • To reinforce the relationship of one category to another. • To represent addition with drawings, expressions, or equations. • To prepare the child for abstract addition.
Small Bead Frame <ul style="list-style-type: none"> • Introduction • Addition • Subtraction • Naming Quantities by Category • Multiplication 	<ul style="list-style-type: none"> • To provide the child with an opportunity to work in a more symbolic way with addition, subtraction, and multiplication. • To offer an opportunity for the child to apply memorized knowledge of the essential combinations. • To reinforce that there are not more than 9 in any category in the decimal system. • To reinforce zero as a place holder. • To reinforce the importance of straight lines when recording columns of numbers. • To offer an opportunity to practice recording quantities.
Decomposing Numbers <ul style="list-style-type: none"> • Teacher-Created Lessons Using: <ul style="list-style-type: none"> • Seguin Boards • Addition Strip Board • Subtraction Strip Board • Golden Beads 	<ul style="list-style-type: none"> • To decompose numbers less than or equal to 10 into pairs. • To find the number that makes 10 when added to a given number. • To compose and decompose numbers from 11 to 19 into tens and ones. • To demonstrate understanding that numbers are composed of tens and ones.
Recording in the Four Operations	<ul style="list-style-type: none"> • To record answers with a drawing or equation.
Problem Solving <ul style="list-style-type: none"> • Teacher-Created or Purchased Cards • Experiences in the Classroom 	<ul style="list-style-type: none"> • To verify the concepts of the operations of the decimal system. • To offer an opportunity to apply the knowledge of the operations to real-life situations. • To give the child another outlet for his explosion into reading and writing. • To solve addition and subtraction word problems by using objects or drawings to represent the problem. • To add and subtract within 10 (using objects or drawings to represent the problem).

ASSESSMENT VOCABULARY

add	number	Cognitive Verbs
addition	ones	act out
compose	pair	answer
decompose/decomposition	represent	compose
equal	subtract	understand
equation	subtraction	decompose
expression	word problem	record
less than (<)		represent
		solve

ASSESSMENT CONSIDERATIONS

Students will be asked to:**Addition**

- Represent addition with objects (Montessori materials), fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations. (K.OA.A.1)
- Record answers with a drawing or equation. (K.OA.A.4)

Subtraction

- Represent subtraction with objects (Montessori materials), fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations. (K.OA.A.1)
- Record answers with a drawing or equation. (K.OA.A.4)

Decomposing Numbers

- Decompose numbers less than or equal to 10 into pairs in more than one way (using objects or drawings). (K.OA.A.3)
- Record each decomposition by a drawing or equation. (K.OA.A.3)
- Find the number that makes 10 when added to a given number (using objects or drawings). (K.OA.A.4)
- Compose and decompose numbers from 11 to 19 into tens and ones (using objects or drawings). (K.NBT.A.1)
- Record composition or decomposition of numbers by a drawing or equation. (K.NBT.A.1)
- Demonstrate understanding that these numbers are composed of tens and ones. (K.NBT.A.1)

Problem Solving

- Solve addition word problems. (K.OA.A.2)
- Add within 10 (using objects or drawings to represent the problem). (K.OA.A.2)
- Solve subtraction word problems and add and subtract within 10 (using objects or drawings to represent the problem). (K.OA.A.2)
- Subtract within 10 (using objects or drawings to represent the problem). (K.OA.A.2)

COMMON CORE STATE STANDARDS (CCSS.MATH.CONTENT)	
K.OA.A.1	Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (claps), acting out situations, verbal explanations, expressions, or equations.
K.OA.A.2	Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.
K.OA.A.3	Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., $5 = 2 + 3$ and $5 = 4 + 1$).
K.OA.A.4	For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.
K.NBT.A.1	Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (such as $18 = 10 + 8$); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.

HEAD START EARLY LEARNING OUTCOMES FRAMEWORK (HELOF) GOALS		
DEVELOPMENTAL PROGRESSION		INDICATORS
36 TO 48 MONTHS	48 TO 60 MONTHS	BY 60 MONTHS
MATHEMATICS DEVELOPMENT (P-MATH)		
OPERATIONS AND ALGEBRAIC THINKING		
P-MATH 6. Child understands addition as adding to and understands subtraction as taking away from.		
<p>Begins to add and subtract very small collections of objects with adult support. For example, the teacher says, “You have 3 grapes and get 1 more. How many in all?” Child counts out 3, then counts out 1 more, then counts all 4: “1, 2, 3, 4. I have 4!”</p>	<p>Solves addition problems by joining objects together and subtraction problems by separating, using manipulatives and fingers to represent objects.</p>	<ul style="list-style-type: none"> • Represents addition and subtraction in different ways, such as with fingers, objects, and drawings. • Solves addition and subtraction word problems. Adds and subtracts up to 5 to or from a given number. • With adult assistance, begins to use counting on from the larger number for addition. For example, when adding a group of 3 and a group of 2, counts “One, two, three...” and then counts on “Four, five!” (keeping track with fingers). When counting back for subtraction such as taking away 3 from 5, counts, “Five, four, three... two!” (keeping track with fingers).

MEMORIZATION

SKILLS INVENTORY

Identifies the essential combinations of math facts in all four operations using the Montessori materials leading to fluency.

MONTESSORI LESSONS

PURPOSES

ADDITION

Addition Snake Game

- Counting the Snake
- Two by Two
- Addition Snake Game Control of Error
- Matching the 10s

- To give the first practice in memorization of the essential combinations in addition.
- To help the child experience the fact that no two quantities less than ten can make more than eighteen.
- To reinforce all of the possible combinations that make ten.
- To offer another way for the child to check his own work.
- To fluently add within 5.

Addition Strip Board

- Exploration
- Using the Tables
- Essential Combinations
- How Many Ways?
- The Commutative Law

- To guide the child through all of the possible essential combinations of addition in a systematic fashion.
- To show that no essential combination exists beyond $9 + 9 = 18$.
- Purpose for the red line on board: to show how quantities greater than 10 are divided into two sections—one section of ten and the second section leading toward another ten.
- To reinforce the mechanism of addition helping the child to see and memorize the essential combinations of addition.
- Using the control charts is how the child will check their work for all the following addition works.
- To introduce the Commutative Law: the order of the addends does not affect the sum.
- To fluently add within 5.

Addition Chart

- Full
- Commutative
- Odd and Even
- Blank

- Further memorization of the essential addition combinations.
- To offer the possibility of repetition.
- To focus the child's attention on the few combinations they still need to memorize, serving as a test of known facts.
- To fluently add within 5.

continues

MONTESSORI LESSONS		PURPOSES
SUBTRACTION		
<p>Subtraction Snake Game</p> <ul style="list-style-type: none"> Counting the Snake Control of Error Two by Two 	<ul style="list-style-type: none"> To familiarize the child with all of the essential combinations in subtraction. To offer the first step toward memorizing the essential combinations in subtraction. Indirectly to prepare the child for algebra: equal quantities of opposite signs cancel each other (this is not something we ever tell the children—it’s just to give them some experience). To fluently subtract within 5. 	
<p>Subtraction Strip Board</p> <ul style="list-style-type: none"> Exploration Essential Combinations How Many Ways? 	<ul style="list-style-type: none"> To practice subtraction, leading to the memorization of the essential combinations in subtraction. To show that subtraction is the opposite action of addition. To fluently subtract within 5. 	
<p>Subtraction Chart</p> <ul style="list-style-type: none"> Practice Blank 	<ul style="list-style-type: none"> Further exploration of the essential combinations in subtraction. To offer the possibility of repetition. To focus the child’s attention on the few combinations that he still needs to memorize, and to serve as a test of known facts. To fluently subtract within 5. 	
MULTIPLICATION		
<p>Multiplication Bead Bars</p> <ul style="list-style-type: none"> Exploring the Tables How Many Ways? Multiplying by 10 	<ul style="list-style-type: none"> To familiarize the child with all of the essential combinations in multiplication. To show the geometric form of multiplication demonstrating that the multiplier is not a solid body—it merely indicates how many times a quantity is repeated—and a succession of lines creates a plane (surface). Preparation for division—helping the child visualize the divisibility of numbers. Indirect preparation for algebra and geometry. To introduce to the child that you just add a zero on the end of any number multiplied by 10. 	
<p>Multiplication Bead Board</p>	<ul style="list-style-type: none"> To practice multiplication leading to memorization of the essential combinations. 	
<p>Multiplication Chart</p> <ul style="list-style-type: none"> Full Commutative Blank 	<ul style="list-style-type: none"> To further memorization of the essential combinations of multiplication. To offer repetition. To focus the child’s attention on the few combinations she still needs to memorize and to offer a test of known facts. 	

continues

MONTESSORI LESSONS	PURPOSES
DIVISION	
Unit Division Board <ul style="list-style-type: none"> • Exploration • Division from 81 • Finding Essential Combinations • Connection to Multiplication 	<ul style="list-style-type: none"> • To familiarize the child with the various ways that quantities can be divided. • To show that not every quantity is evenly divisible, and to show that some quantities are divisible only by a few numbers. • To discover the essential combinations of division. • To practice division leading to the memorization of the essential combinations of division. • To see the relationship between multiplication and division.
Division Chart <ul style="list-style-type: none"> • Practice • Blank 	<ul style="list-style-type: none"> • To further memorize the essential combinations in division. • To offer the possibility of repetition. • To focus the child's attention on the few combinations he still needs to memorize, serving as a test of known facts.

ASSESSMENT VOCABULARY

add
subtract

ASSESSMENT CONSIDERATIONS

Students will be asked to:

- Fluently add within 5. (K.OA.A.5)
- Fluently subtract within 5. (K.OA.A.5)

HEAD START EARLY LEARNING OUTCOMES FRAMEWORK (HELOF) GOALS

There are no Head Start Early Learning Outcomes for Whole Number: Memorization in Primary.

COMMON CORE STATE STANDARDS (CCSS.MATH.CONTENT)

K.OA.A.5	Fluently add and subtract within 5.
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CHAPTER 4

FRACTIONS

FRACTIONS

SKILLS INVENTORY

Demonstrates understanding of the language of fractions (quantities <1) and the concept of fractions as the equal parts of a whole.

Performs all four operations with fractions using Montessori materials.

MONTESSORI LESSONS	PURPOSES
Foundations and Equivalence <ul style="list-style-type: none">• Making Designs• Introduction to Quantity• Naming the Fractions (Introduction to Symbols)• Substitution (Equivalence)	<ul style="list-style-type: none">• To give the child a beginning understanding of the nature of fractions.• To discover the relationships between fractions and whole numbers.
Operations (Sensorial) <ul style="list-style-type: none">• Addition• Subtraction• Multiplication• Division	<ul style="list-style-type: none">• To give the child experience using the four operations with fractions.

ASSESSMENT VOCABULARY & ASSESSMENT CONSIDERATIONS

These skills are not formally assessed in the Primary

COMMON CORE STATE STANDARDS (CCSS.MATH.CONTENT)

There are no Kindergarten Standards for fractions.

HEAD START EARLY LEARNING OUTCOMES FRAMEWORK (HELOF) GOALS

There are no Head Start Early Learning Outcomes for fractions.

CHAPTER 5

MONEY

MONEY

SKILLS INVENTORY

Demonstrates understanding of money by identifying and naming coins (penny, nickel, dime, quarter) and counting the value of coins to \$1.00.

MONTESSORI LESSONS

PURPOSES

Sorting and Matching Money

- To associate the names, shapes, and colors of coins through muscular and visual memory.
- To classify objects into given categories, count the numbers of objects in each category, and sort the category.

Counting Money up to \$1

- To count the value of coins.

ASSESSMENT VOCABULARY & ASSESSMENT CONSIDERATIONS

These skills are not formally assessed in the Primary.

COMMON CORE STATE STANDARDS (CCSS.MATH.CONTENT)

There are no Kindergarten Standards for money.

HEAD START EARLY LEARNING OUTCOMES FRAMEWORK (HELOF) GOALS

There are no Head Start Early Learning Outcomes for money.

INDEXES

STANDARDS TO MONTESSORI INDEX

COMMON CORE STATE STANDARDS STRANDS, DIVISIONS, AND STANDARDS CCSS.MATH.PRACTICE		MONTESSORI CHAPTERS AND SECTIONS
MATHEMATICAL PRACTICE (MP)		
PROBLEM SOLVING		
MP1	Make sense of problems and persevere in solving them.	Mathematical Practice • Problem Solving
MP6	Attend to precision.	
COMMUNICATION: REASONING AND EXPLAINING		
MP2	Reason abstractly and quantitatively.	Mathematical Practice • Communication: Reasoning and Explaining
MP3	Construct viable arguments and critique the reasoning of others.	
MODELING AND USING TOOLS		
MP4	Model with mathematics.	Mathematical Practice • Modeling and Using Tools
MP5	Use appropriate tools strategically.	
SEEING STRUCTURES AND GENERALIZING		
MP7	Count forward beginning from a given number within the known sequence (instead of having to begin at 1).	Mathematical Practice • Seeing Structures and Generalizing
MP8	Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects).	

continues

COMMON CORE STATE STANDARDS STRANDS, DIVISIONS, AND STANDARDS		MONTESSORI CHAPTERS AND SECTIONS
COUNTING AND CARDINALITY (CC)		
KNOW NUMBER NAMES AND THE COUNT SEQUENCE.		
K.CC.A.1	Count to 100 by ones and by tens.	The Decimal System • Number and Quantity Greater than Ten
K.CC.A.2	Count forward beginning from a given number within the known sequence (instead of having to begin at 1).	The Decimal System • Number and Quantity Greater than Ten • Number and Quantity One to Ten
K.CC.A.3	Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects).	
COUNT TO TELL THE NUMBER OF OBJECTS		
K.CC.B.4	Understand the relationship between numbers and quantities; connect counting to cardinality.	The Decimal System • Number and Quantity Greater than Ten
K.CC.B.4.A	When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object.	The Decimal System • Number and Quantity Greater than Ten • Number and Quantity One to Ten
K.CC.B.4.B	Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.	
K.CC.B.4.C	Understand that each successive number name refers to a quantity that is one larger.	
K.CC.B.5	Count to answer “how many?” questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1-20, count out that many objects.	

continues

COMMON CORE STATE STANDARDS STRANDS, DIVISIONS, AND STANDARDS		MONTESSORI CHAPTERS AND SECTIONS
COMPARE NUMBERS		
K.CC.C.6	Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies. ¹	The Decimal System <ul style="list-style-type: none"> • Number and Quantity Greater than Ten
K.CC.C.7	Compare two numbers between 1 and 10 presented as written numerals.	The Decimal System <ul style="list-style-type: none"> • Number and Quantity Greater than Ten • Number and Quantity One to Ten
NUMBERS AND OPERATIONS IN BASE 10 (NBT)		
WORK WITH NUMBERS 11-19 TO GAIN FOUNDATIONS FOR PLACE VALUE		
K.NBT.A.1	Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (such as $18 = 10 + 8$); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.	The Decimal System <ul style="list-style-type: none"> • Number and Quantity Greater than Ten
OPERATIONS AND ALGEBRAIC THINKING (OA)		
UNDERSTAND ADDITION AS PUTTING TOGETHER AND ADDING TO, AND UNDERSTAND SUBTRACTION AS TAKING APART AND TAKING FROM		
K.OA.A.1	Represent addition and subtraction with objects, fingers, mental images, drawings, ¹ sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.	Whole Number Operations <ul style="list-style-type: none"> • Passages to Abstraction Decimals <ul style="list-style-type: none"> • Operations

continues

COMMON CORE STATE STANDARDS STRANDS, DIVISIONS, AND STANDARDS		MONTESSORI CHAPTERS AND SECTIONS
K.OA.A.2	Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.	Whole Number Operations • Passages to Abstraction
K.OA.A.3	Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., $5 = 2 + 3$ and $5 = 4 + 1$).	
K.OA.A.4	For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.	
MEASUREMENT AND DATA (MD)		
DESCRIBE AND COMPARE MEASURABLE ATTRIBUTES		
K.MD.A.1	Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object.	Education of the Senses • Auditory Sense • Olfactory Sense • Gustatory Sense • Visual Sense • Sense of Touch
K.MD.A.2	Directly compare two objects with a measurable attribute in common, to see which object has “more of”/“less of” the attribute, and describe the difference. For example, directly compare the heights of two children and describe one child as taller/shorter.	
CLASSIFY OBJECTS AND COUNT THE NUMBER OF OBJECTS IN EACH CATEGORY.		
K.MD.B.3	Classify objects into given categories; count the numbers of objects in each category and sort the categories by count	Education of the Senses • Auditory Sense • Olfactory Sense • Gustatory Sense • Visual Sense • Sense of Touch

continues

COMMON CORE STATE STANDARDS STRANDS, DIVISIONS, AND STANDARDS		MONTESSORI CHAPTERS AND SECTIONS
GEOMETRY (G)		
IDENTIFY AND DESCRIBE SHAPES		
K.G.A.1	Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as above, below, beside, in front of, behind, and next to.	Education of the Senses <ul style="list-style-type: none"> • Mixed Impressions • Visual Sense • Stereognostic Sense
K.G.A.2	Correctly name shapes regardless of their orientations or overall size.	
K.G.A.3	Identify shapes as two-dimensional (lying in a plane, “flat”) or three-dimensional (“solid”).	
ANALYZE, COMPARE, CREATE, AND COMPOSE SHAPES.		
K.G.B.4	Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/“corners”) and other attributes (e.g., having sides of equal length).	Education of the Senses <ul style="list-style-type: none"> • Mixed Impressions • Visual Sense • Stereognostic Sense
K.G.B.5	Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes.	
K.G.B.6	Compose simple shapes to form larger shapes. For example, “Can you join these two triangles with full sides touching to make a rectangle?”	

HEAD START EARLY LEARNING OUTCOMES FRAMEWORK		MONTESSORI CHAPTERS AND SECTIONS
MATHEMATICS DEVELOPMENT (P-MATH)		
COUNTING AND CARDINALITY		
P-MATH-1	Child knows number names and the count sequence.	The Decimal System <ul style="list-style-type: none"> • Number and Quantity Greater than Ten • Number and Quantity One to Ten
P-MATH-2	Child recognizes the number of objects in a small set.	The Decimal System <ul style="list-style-type: none"> • Number and Quantity One to Ten
P-MATH-3	Child understands the relationship between numbers and quantities.	
P-MATH-4	Child compares numbers.	
P-MATH-5	Child associates a quantity with written numerals up to 5 and begins to write numbers.	
OPERATIONS AND ALGEBRAIC THINKING		
P-MATH-6	Child understands addition as adding to and understands subtraction as taking away from.	Whole Number Operations <ul style="list-style-type: none"> • Passages to Abstraction Decimals <ul style="list-style-type: none"> • Operations

MONTESSORI TO STANDARDS INDEX

CHAPTER AND SECTION	STANDARDS ALIGNED	
MATHEMATICAL PRACTICE		
Human Tendencies and the Mathematical Mind	CCSS.MATH	
	No Standards Aligned	
Problem Solving	CCSS.MATH.PRACTICE	
	MP	Mathematical Practice <ul style="list-style-type: none"> • Problem Solving
Communication: Reasoning and Explaining	CCSS.MATH.PRACTICE	
	MP	Mathematical Practice <ul style="list-style-type: none"> • Communication: Reasoning and Explaining
Modeling and Using Tools	CCSS.MATH.PRACTICE	
	MP	Mathematical Practice <ul style="list-style-type: none"> • Modeling and Using Tools
Seeing Structures and Generalizing	CCSS.MATH.PRACTICE	
	MP	Mathematical Practice <ul style="list-style-type: none"> • Seeing Structures and Generalizing
THE DECIMAL SYSTEM		
Number and Quantity One to Ten	CCSS.MATH.CONTENT	
	CC	Counting and Cardinality <ul style="list-style-type: none"> • Know number names and the count sequence • Count to tell the number of objects • Compare numbers
	HELOF COGNITION	
	P-MATH	Mathematics Development <ul style="list-style-type: none"> • Counting and Cardinality

continues

CHAPTER AND SECTION	STANDARDS ALIGNED	
Number and Quantity Greater than Ten	CCSS.MATH.CONTENT	
	CC	Counting and Cardinality <ul style="list-style-type: none"> • Know number names and the count sequence • Count to tell the number of objects • Compare numbers
	NBT	Numbers and Operations in Base 10 <ul style="list-style-type: none"> • Work with numbers 11-19 to gain foundations for place value
	HELOF COGNITION	
	P-MATH	Mathematics Development <ul style="list-style-type: none"> • Counting and Cardinality
WHOLE NUMBER OPERATIONS		
Operations	CCSS.MATH.CONTENT	
	OA	Operations and Algebraic Thinking <ul style="list-style-type: none"> • Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from
	HELOF COGNITION	
	P-MATH	Mathematics Development <ul style="list-style-type: none"> • Counting and Cardinality
Passages to Abstraction	CCSS.MATH.CONTENT	
	OA	Operations and Algebraic Thinking <ul style="list-style-type: none"> • Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from
	NBT	Numbers and Operations in Base 10 <ul style="list-style-type: none"> • Work with numbers 11-19 to gain foundations for place value
	HELOF COGNITION	
	P-MATH	Mathematics Development <ul style="list-style-type: none"> • Operations and Algebraic Thinking

continues

CHAPTER AND SECTION		STANDARDS ALIGNED
Memorization	CCSS.MATH.CONTENT	
	OA	Operations and Algebraic Thinking <ul style="list-style-type: none"> Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from
FRACTIONS		
Fractions	CCSS.MATH.CONTENT	
	Common Core Fractions Standards begin in 1st grade	
MONEY		
Money	CCSS.MATH.CONTENT	
	Common Core Money Standards begin in 2nd grade	

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